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Numerical evaluation of fracture behavior of split concrete beams reinforced with steel sheet

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Abstract

Since the parameters affecting the strengthening of cracked beams and their failure behavior after strengthening are very important, this research is devoted to the numerical investigation of the failure behavior of cracked concrete beams reinforced with steel sheet. The investigated beams have an initial crack in the middle of the span and are reinforced with a steel layer under the beam. Using the finite element method and non-linear static analysis, the bearing capacity of the samples, the growth and opening of the crack opening have been evaluated. First, in order to evaluate the accuracy of the modeling, the results obtained with the data obtained from the existing laboratory work were compared, and after ensuring the correctness of the modeling, the effect of changing various parameters, including sheet thickness, mechanical characteristics of concrete, initial crack length, was investigated. The results show that the load-opening diagrams of the crack openings have two maximum load points, the first and second maximum points increase with the increase of concrete strength and sheet thickness, and due to the increase in the initial crack length, the first maximum point decreases, while the maximum point The second remains almost unchanged. (DOI:https//**doi.org/10.52547/JCER.4.3.20**)

Keywords: cracked concrete beam, steel sheet, crack opening, nonlinear finite element method;

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1. Introduction

Nowadays, strengthening of concrete structures with the help of FRP is used as a new method in the field of strengthening. Using FRP for strengthening compared to traditional methods has advantages such as increased strength and hardness, light weight, good corrosion resistance, low thickness, easy to carry and install. After installing the sheet to the concrete beam, the response of the structure, including ductility, strength and failure parameters, is different compared to the behavior of the unreinforced structure. Therefore, experimental tests, analytical and numerical methods are necessary to predict the behavior of structures after retrofitting. The presence of cracks in structures is inevitable, so regardless of its effect, the safety of the structure may be compromised. On the other hand, crack growth due to repeated loading that is generally applied to structures is an issue that, without considering it in the design, it is possible to significantly reduce the useful life of the structure. In recent years, a number of researchers investigated the nature of the separation mechanism between FRP sheet and concrete by using fracture mechanics and determining the obtained energy [1-4]. Taljesten [5] presented a linear equation to calculate the bearing capacity of reinforced concrete with FRP sheet. under tensile axial load and based on it, the maximum load and ultimate load is a function of fracture energy (GF), modulus of elasticity and thickness of FRP sheet

Yashiza et al. [6, 7] succeeded in determining the fracture energy and the stress-strain relationship by conducting a one-way shear test on reinforced concrete samples with FRP sheets. Wu and Yen [8] studied the behavior of cracking and failure due to separation of reinforced concrete beams reinforced with FRP sheets. By performing finite element analysis and by examining different types of separation development during the interaction between concrete and FRP sheet and crack distribution in concrete, they found that the properties of adhesive and concrete have the greatest effect on the types of separation development and crack distribution. Also, the bond strength, fracture energy of adhesive layer, tensile strength and fracture energy of concrete were fully investigated and the effect of these parameters on concrete cracking behavior, structural bearing capacity and types of separation were studied. In order to postpone the separation of the sheet from the concrete surface, different solutions have been presented and investigated experimentally and by simulation. Shahbaz Panahi et al [9], by presenting a numerical method, investigated the shear strengthening of reinforced concrete beams using FRP sheets. They focused their studies on determining the length of FRP failure zone, energy release rate and crack propagation path. The results show that the failure mode in the existing model and the control beam (without reinforcement) is due to diagonal shear cracks. Also, the failure mode of the reinforced sample in the laboratory is concrete shear rupture and sheet separation, while the failure of the corresponding modeled sample is It is due to the rupture of the steel sheet in the opening of the cut Considering that FRP sheets can delay the crack development and reduce the width of the crack opening in concrete, many researches [12-10] analytically presented relationships to predict the bearing capacity of cracked reinforced concrete beams. In this regard, Wu and Davids [13], considering the non-slip between concrete and FRP sheet, found that the crack growth stops when its length increases to a certain value. A number of researchers investigated the resistance of concrete samples reinforced with FRP sheets to peeling by performing peeling tests. They considered the effect of different types of FRP sheet and glue, the way of preparing the concrete surface and the strength of concrete on the flaking of the samples and by determining the fracture energy by analytical method, they found that the fracture energy of the contact surface of concrete and FRP sheet and the hardness of the sheets in the flaking of the samples should be be considered[14] High stresses may cause cracks to form near the contact surface of concrete and FRP, and if the necessary energy is provided for their expansion, these cracks will progress. Recently, nonlinear fracture mechanics models have been presented [17-15], and the results of one-way shear tests were used to determine the fracture parameters. In any case, due to the difference between the failure mode of the reinforced beams and the one-way cut test samples, these methods cannot be used to determine the exact failure parameters.

Katz [18], in order to investigate the stress transfer mechanism between reinforced concrete beams and steel sheets and GFRP, 36 beams with integrated concrete cross-section and with a weak concrete top, which indicates the damaged surface of the concrete, were built and recorded and analyzed the spread of strains in concrete and FRP layers. has done. The results show that the presence of a weak concrete layer on the surface does not have much effect on the bearing capacity, and the type of FRP has the greatest effect on the bearing capacity, and in general, it can be said that increasing the strength of the concrete core of the samples with a weak top layer of concrete increases the bearing capacity. .

Many researches have been done on the behavior of FRP-reinforced concrete members, but what is important is the proper amount and manner of strengthening the members after cracking with FRP sheets. In this context, Wu et al.[19] investigated the fracture of concrete beam reinforced with FRP sheet using analytical method and compared the results with what was obtained from the three-point bending test of reinforced beams. They concluded that in the early stages of loading, the P-CMOD relationship is linear, but after the first crack, it is nonlinear. In fact, after the amount of load reaches its first maximum value, it decreases and the crack opening opens. In the continuation of loading, due to the combined performance of concrete and FRP sheet, the amount of load increases until it reaches its second maximum point. They attribute the fluctuations observed in the crack opening load-distension curves to the different adhesion conditions between the FRP sheet and concrete at different points.

Achinta and Borgon [20] used the concepts of fracture mechanics based on total energy balance to determine the load under which the FRP sheet separates from the concrete beam. They found that the only important and effective parameter is the fracture energy of the contact surface of concrete and FRP sheet, and the development of separation occurred in the concrete area between the sheet and tensile steel bars, and also the presence of steel bars prevents the development of the fracture area. The results show that the deviation of the FRP sheet force from the crack opening causes tensile stresses in the crack opening and for this reason the crack starts to progress.

Mohammadi et al.[21] performed a one-way shear test to investigate the effective parameters on the interaction zone of concrete and steel sheet and also investigated the effect of boundary conditions on the resistance of the interaction zone with the help of numerical analysis. In another study [22], by combining the adhesive crack method and the damaged concrete model, they simulated the behavior of FRP reinforced concrete beams using the advanced finite element method. In order to verify the results of the modeling, they also investigated the behavior of these beams in a laboratory. By studying how the stress spreads in the location of the main cracks, they investigated the separation of the sheet from the concrete surface in samples with a variable crack position relative to the middle of the opening and found that the separation of the sheet from the tip of the diagonal cracks and near the main bending-shear cracks where the ratio of anchor- The cut is high, it begins.

As mentioned earlier, most of the studies have been done in laboratory and analytical form around beams reinforced with FRP sheets. By using the finite element method, it is possible to make a more complete investigation about the behavior of the structures that have become resistant after cracking. Due to the high costs of laboratory research, it is appropriate to use the finite element method to more accurately estimate the behavior of these structures. The main goal of the current research is numerical modeling of concrete beams with initial crack reinforced with steel sheet using the finite element method. In this research, more details about the modeling of the samples have been discussed, including how to model the glue, which has rarely been considered in previous studies. Also, by comparing the results of modeling and laboratory results done by other researchers and ensuring the accuracy of the modeling, the parameters affecting the failure behavior of these beams after strengthening were evaluated with the help of numerical method.

2- Numerical modeling

In order to numerically analyze the concrete beams with initial crack reinforced by steel sheet, using the finite element method and ABAQUS software, a model of the laboratory sample that was implemented and tested under static load by Wu et al. [19] has been prepared and after comparing the results of the numerical model of this research and the aforementioned laboratory results and ensuring the correctness of the numerical model, the results are presented for other numerical analysis modes.

• Laboratory sample of reinforced steel beam with initial crack

In 2010 [19], ææ et al. analyzed the behavior of concrete beams with primary cracks reinforced by a layer of steel sheet under the beam analytically, numerically and experimentally. Figure 1 shows the characteristics of the beam with initial crack under three-point bending. where (b) beam width, (h) beam height, (2L) span length, (a0) initial crack length, (ha) adhesive thickness, (hp) steel sheet thickness, (bp) sheet width, (P) The applied load (2Ld) is the length of the area of the sheet that is not connected to the concrete to prevent diagonal cracking during loading.

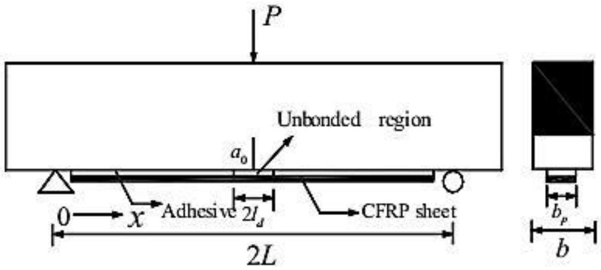

Figure 1: Concrete beam with primary crack and [19] steel sheet.

The beams were made in three groups with different heights. For the bending failure direction of the samples, the ratio of the length of the opening to the height of beam 4 is selected, the width and the ratio of the length of the initial crack to the height of the samples (a0/h) are constant and equal to 0.3 and 150 mm, respectively. While the height is variable and three heights of 200, 250 and 300 mm have been selected. Based on the results of the tests, the mechanical characteristics of the concrete used are according to Table 1 and the characteristics of the steel sheet are also according to Table 2.

Table 1: Mechanical properties of used concrete [19].

Tensile strength (MPa)	Modulus of rupture (MPa)	Modulus of elasticity (GPa)	Poisson's ratio
28.50	23.3	45.29	248.0

The settings required to measure the bearing capacity and the crack opening of the samples under the three-point bending test are shown in Figure 2.

Table 2: Geometrical and mechanical characteristics of [steel] sheet

Sheet type	thickness (mm)	Modulus of elasticity (GPa)	Tensile strength (MPa)	Ultimate strain
CFRP	167.0	240	3916	0017.0

Figure 2: Laboratory settings for loading and recording sample data [19].

• Numerical modeling of slotted beam reinforced with steel sheet

Abaqus software version 14.6 was used to investigate the fracture behavior of the beams and the initial crack growth in the samples using the finite element method. Solid and shell elements were used in 3D modeling of concrete beam and XFEM method was chosen for crack modeling according to material properties. The criterion for crack opening is the maximum stress, so that the crack expands when the maximum stress exceeds the tensile strength of concrete

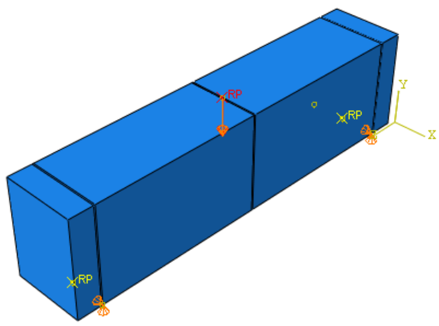

Figure 4: Beam model with initial crack and support conditions

For proper meshing, the results obtained from the analysis of the simulated samples in the software and the corresponding laboratory samples were compared with each other and by observing the convergence of the responses based on the sensitivity analysis, the size and density of the elements were selected. Also, according to Figure 5, the size of the mesh is smaller and larger in the insensitive areas due to the sensitivity of the elements to the crack growth around it.

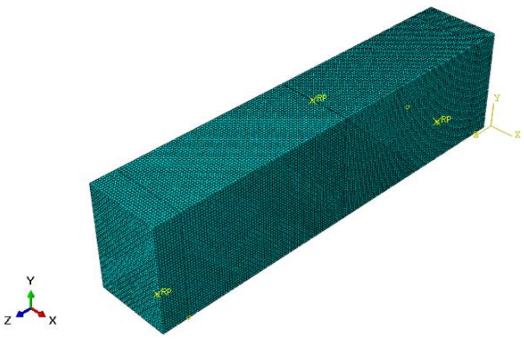

Figure 5: Finite element model meshing

3- Comparison of laboratory results and numerical analysis

The investigated beams consist of three parts: concrete, glue and steel sheet. Since the resistance of each section has an effect on the mode of failure and the rate of crack development, four stages can be considered in loading until the final failure. At first, the slip between the concrete and the sheet is insignificant, and with the application of force, the sheet is stretched in the middle of the opening and a crack develops. In this stage, separation did not occur and simultaneously with the progress of the crack, the load reaches the initial maximum point (P1max). In the next stage, while the microcracks develop and the width of the crack also increases, the bearing capacity decreases. Then in the third stage, with opening the opening of the crack, tensile stresses are added in the steel sheet and at this time the effect of the sheet affects the bearing capacity and causes the load to reach the second maximum point (P2max). Finally, the concentration of shear stresses in the vicinity of the middle of the opening along with the development of a vertical crack, Horizontal micro-cracks also increased on the common surface of concrete and sheet, leading to the separation of the sheet from the concrete and finally, the complete failure of the sample.

Wu et al. [19] numerically investigated this process and obtained the load-displacement diagrams of the crack opening. In Figure 6-A, B, and C, the P-CMOD diagram obtained from the experimental and numerical results [19], for beams with a height of 200, 250, and 300 mm, respectively, is compared with the results obtained in the present study using Abaqus software. As can be seen, the process of changes in the diagrams obtained based on numerical and laboratory methods are similar. So that at first the load changed linearly in relation to the opening of the crack opening and after the development of the crack, the behavior became non-linear, after reaching the P1max point, the bearing capacity decreased and again due to the presence of the steel sheet, the capacity increased until it reached the P2max point.

Although the amount of load reduction in the laboratory is lower than the corresponding value obtained from the modeling, what is more important is the increase in capacity due to the presence of the sheet, because in the transition between the primary and secondary maximum points, the stress transfer takes place between the concrete and the sheet.

According to the results of the difference between P1max and P2max obtained from the laboratory, numerical and modeling results in this research, it is less than ten percent. The fluctuations in the graph obtained from the laboratory investigation can also be attributed to the non-uniformity of the conditions of sticking the sheet to the concrete surface along the span of the beam. The rupture of the tested samples happened due to the separation of the sheet from the concrete surface and the development of the crack [19]. In this way, the separation of the sheet started from the vicinity of the initial crack and with the progress of the crack, it moves towards the support. In figure 7-a and b, the failure mode of the laboratory sample is shown.

According to Figure 7, it can be expected that the maximum stresses in the sheet occur near the crack. In the numerical example, it can be seen that the maximum shear stresses in the steel sheet occurred near the middle of the opening, which caused the separation of the sheet from this area, and the amount of stress decreases with the distance from the initial crack location. Figure 8 shows the connection of steel sheet to concrete beam. According to the investigations, there is a relatively good agreement between the numerical and laboratory models in different aspects. Therefore, it is possible to trust the results of numerical analyzes with the help of finite elements regarding the modeling of concrete, glue and steel sheet.

4- Parametric studies

After ensuring the validity of the numerical modeling using the finite element method, parametric analyzes have been performed for the steel plate reinforced

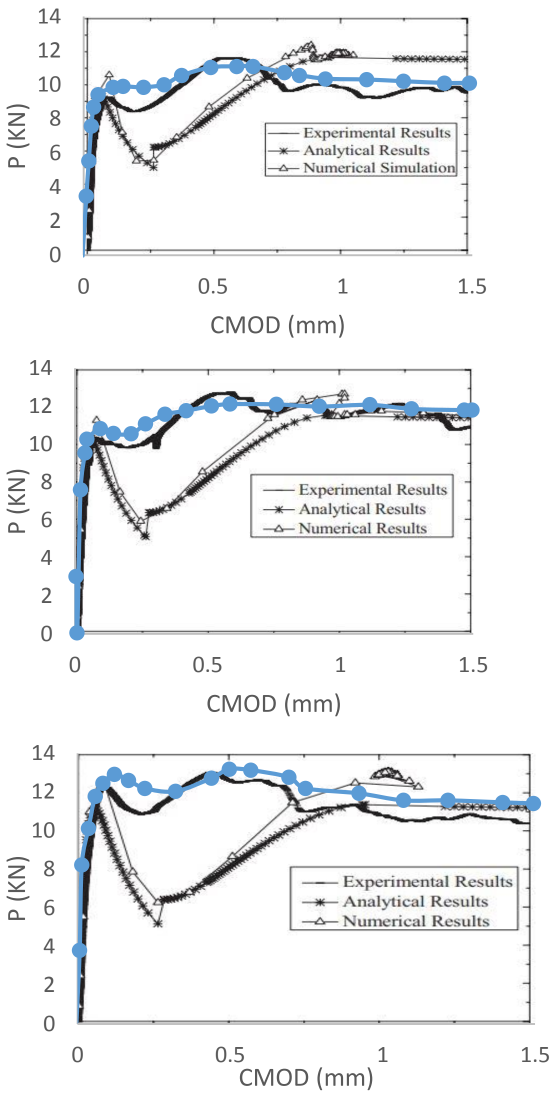

Figure 6: Comparison of P-CMOD diagrams of reference [19] and Abaqus: a) mm200h=, b) mm250h=c) mm=300h

(a)

(b)

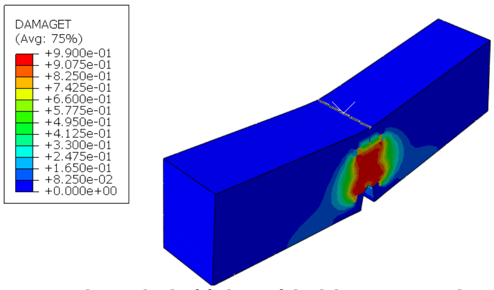

Figure 7: The method of failure of the laboratory sample a) Beam failure

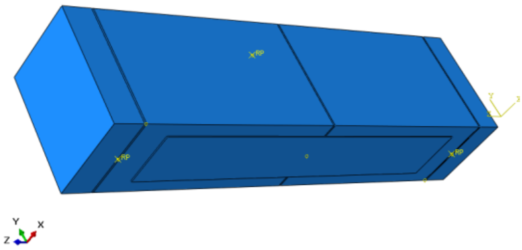

Figure 8: Connecting steel sheet to concrete beam

split beam by changing the mechanical properties of concrete, the thickness of the steel plate, the ratio of the initial crack length to the beam height, and the adhesive strength. In all analyses, the type of elements and the method of modeling are the same as those considered in the validation section

• The effect of changing the mechanical properties of concrete

In order to investigate the effect of changing the compressive strength of concrete on the fracture behavior of steel-reinforced beams, the geometric and mechanical characteristics of the sheet have been selected according to Table 2. By changing the compressive strength of concrete, other mechanical properties will also change.
This diagram is determined based on the results of the uniaxial compression test of concrete. For concrete under pressure, three areas of the diagram are introduced. The first part of the diagram is assumed to be elastic up to the corresponding limit stress. The amount of this stress is considered equal [4]. where is the compressive strength of concrete. Strain is equal to stress. Young's modulus is also calculated based on [4] and Poisson's ratio is considered equal. The second part of the diagram, which has a parabolic shape, starts from the point with the limit stress and continues until reaching the highest compressive strength of concrete. This part of the diagram is determined by the equation (1):

$$\sigma_c = \left(\frac{kn-n^2}{1+(k-2)n}\right) f_{ck} \qquad (1)$$

$$\varepsilon_{c1} = 0.0022 \quad k = 1.1E_{cm} \times \frac{\varepsilon_{c1}}{f_{ck}} \quad n = \frac{\varepsilon_c}{\varepsilon_{c1}} \qquad (2)$$

where is the modulus of elasticity of concrete.
The third part of the stress-strain curve is the descending part of the graph from to, where the reduction factor is considered equal to 0.85. The final strain of concrete ε_{cu} at rupture rf_c is equal to 0.01 stress.
The load-opening diagram of the crack opening for three cases of concrete with a resistance of 55, 45, and 35 MPa is presented in Figure 11.
According to the above figure, it can be seen that with the increase of concrete strength, the primary and secondary maximum points increase. The rate of increase for the initial maximum point of the sample with 55 MPa resistance compared to the sample with 35 MPa resistance is equal to 14% and for the second maximum point of the said samples is equal to 1.2%. This increase can be justified considering that the bond stress in the damaged area around the crack (FPZ) as well as the shear strength in the contact surface of concrete and sheet according to equation (1) are directly proportional to the tensile strength of concrete.
Since the maximum shear stress in the steel sheet occurs around the crack, according to Figure 12, the strain of the elements is also higher in this area and

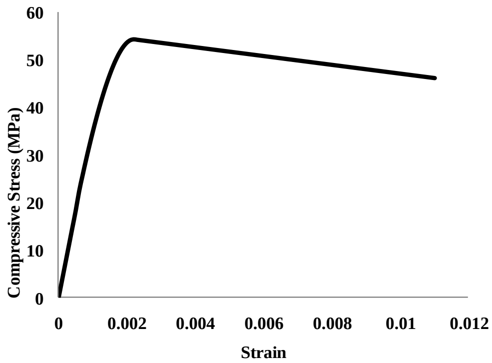

Figure 9 - Tensile stress-strain curve for concrete with a compressive strength of 55 MPa

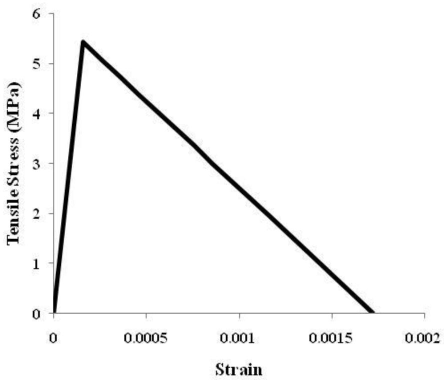

Figure 10 - Stress-strain curve for concrete with a compressive strength of 55 MPa

Table 3: Mechanical properties of concrete in numerical analysis.

pushing resistance (MPa)	Modulus of elasticity (MPa)
35	28465
45	30951
55	33442

gradually decreases as they move away from the crack.

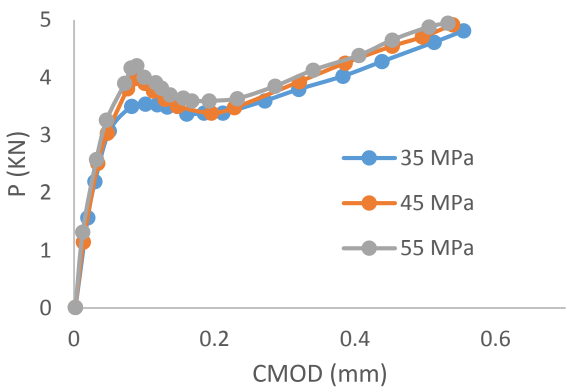

Figure 11: Load-strain diagram of crack opening for beams with resistance of 35, 45 and 55 MPa.

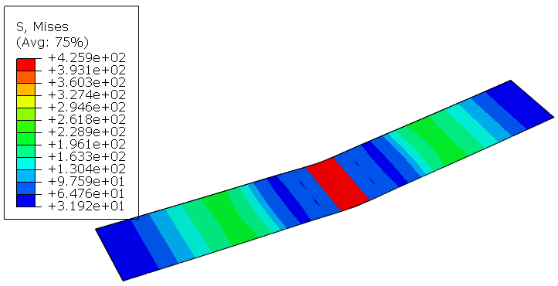

Figure 12: Stress distribution in steel sheet

• The effect of changing the thickness of the steel sheet compared to CFRP

Since the geometry of the steel sheet can be one of the influencing factors on the fracture behavior of concrete beams, the effect of changing the thickness of the sheet is investigated in this section. In this study, the geometric characteristics of the beam are fixed, the characteristic strength of concrete is 40 MPa, the crack length is 100 mm, and with other parameters remaining constant, the sheet thickness is 1.0, 15.0, and 2.0 mm. The load-displacement diagram of the crack opening is according to Figure 12.

The primary and secondary maximum load points in the diagram increase with the increase of sheet thickness. Because a sheet with a greater thickness

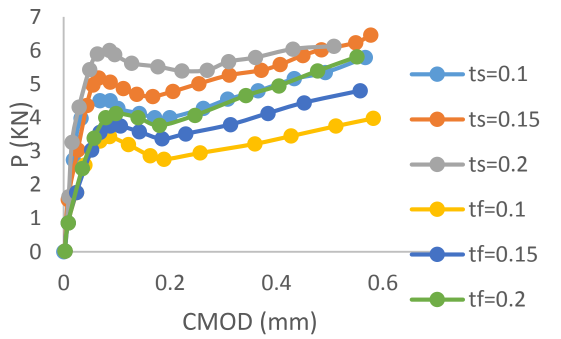

Figure 12: Load-expansion diagram of crack opening for variable thickness of CFRP sheet and steel sheet

can withstand a greater tensile force, and with an increase in thickness from 1.0 to 2.0, the primary maximum point will increase by 35% and the secondary by 19%. Therefore, it can be said that the second maximum load is more sensitive to the sheet thickness. The changes of stress along the length of the sheet with the change of its thickness are shown in Figure 13. Also, the use of steel sheet compared to CFRP sheet has shown higher initial hardness and more resistance by about 30% compared to samples with similar thickness in CFRP sheet.

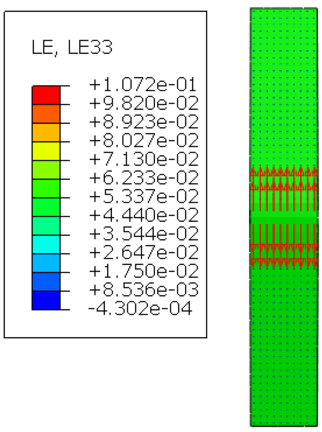

Figure 13: Strain changes along the length of the steel sheet

According to the above figure, it can be seen that in the length of the sheet which is connected to the concrete and their performance is integrated, the strains in the middle part have the highest values and by moving towards the two ends, the strains show almost zero.

• The effect of changing the initial crack length

In order to investigate the effect of the length of the initial crack on how the beam breaks as well as its bearing capacity, other parameters were considered constant for three crack lengths of 80, 100, and 120 mm. The characteristic strength of concrete is 40 MPa and the specifications of the sheet are also according to table 2. The changes in the bearing capacity of the samples in relation to the opening of the crack opening are shown in Figure 14.

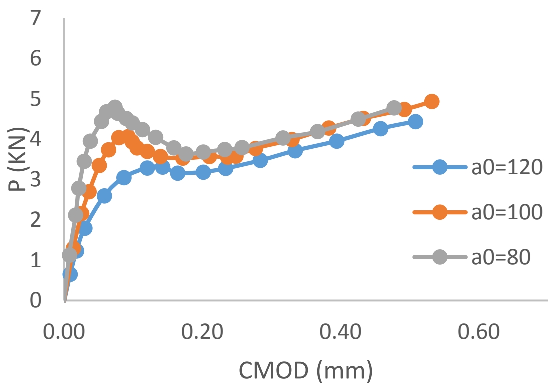

Figure 14: Load-discharge diagram of crack opening for variable initial crack length

Based on table 5, it can be seen that the reduction of the bearing capacity of the beam by changing the length of the crack from 80 to 120 mm is about 30%.

Table 5- Maximum bearing capacity of samples with different crack lengths

Thickness of steel sheet**(**mm**)**	P1max (kN)	P2max (kN)
80	3.3	4.4
100	4.05	4.93
120	4.78	4.78

It can be seen from Figure 14 that with the increase of the crack length, the primary maximum load decreases and the secondary maximum load remains almost unchanged. The reason for this behavior can be the reduction of the bearing capacity of the beam due to the increase in the length of the crack. Because the first maximum point in the load-opening diagram of the crack opening is related to concrete failure due to crack development and the second maximum point is due to steel tensile force.

5- Conclusion

In this research, the behavior of concrete beams with primary crack reinforced with steel sheet was investigated by changing the effective parameters using the finite element method, and the following results were obtained:

- The comparison of numerical and experimental results shows that the numerical modeling of the behavior of cracked beams reinforced with steel plate by the finite element method has a good accuracy.

- Rupture of the samples occurs by separating the steel sheet from the place of the crack, and its development towards the end of the sheet simultaneously with the progress of the crack.

- According to the relationship between the characteristic strength of concrete and its other mechanical properties, with the increase of the compressive strength and then the tensile strength, the bearing capacity of the samples increases.

- By increasing the thickness of the sheet, the bearing capacity and tensile stress of the steel sheet increase in the area connected to the concrete, and the amount of stress decreases around the crack.

- Changing the length of the initial crack has a significant effect on the initial maximum load, and with the increase of the length of the crack, the load-carrying capacity is significantly reduced.

- In general, according to the obtained results, it can be seen that the length of the primary crack is the most effective factor on the primary and secondary maximum load in the load-opening diagrams of the crack opening.

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